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Electroacoustic radiation characteristics of microstrip phased array antennas in plasma medium

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Abstract : The three important radiation properties *viz.* radiation conductance, directive gain and radiation efficiency are studied for three different possible cases of microstrip phased array antennas in ionized plasma medium The two cases *i.e* linear and planar array antennas are common but the third case *i.e* circular array is new and has certain advantages over the other two. The results obtained for this array antenna are compared with two earlier array antennas and with the results obtained for two element array by Saxena *et al* (1989) It is concluded that the presence of plasma greatly modified the radiation properties The antennas exhibit almost zero response when they are surrounded by fully ionized plasma.

Keywords : Microstrip phased array antenna, plasma medium, radiation properties

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1. Introduction

The phased array antennas find wide applications in the fields where direction-dependent beams are required. The antenna beam is electronically scanned by changing the phase gradient across the array. This leads to minimise the side lobe level while maintaining the main lobe gain. Electronically controlled phase shifters like pin-diode phase shifters, ferrite phase shifters *etc.* are generally used in the beam forming network of a phased array [1–3]. It is well known that micorstrip array antennas have several profits in comparison to the conventional array antennas [4–6]. It has also been estabished that when these antennas are mounted on-board aerospace vehicles, they encounter plasma media. Consequently, electroacoustic waves are also generated in addition to usual electromagnetic waves which alter radiation properties significantly [6–9].

In the present paper, a comparative study has been made to analyse the radiation properties like radiation conductance, directive gain and radiation efficiency of three different 4-element array antennas *viz.* linear array antenna, planar array antenna and circular array antenna. Most of the earlier workers have considered only two possibilities for the arrangements of elements, either along a single direction (linear array) or arranged in a plane (planar array). The third possibility is to arrange the elements along a circular ring. This leads to the concept of microstrip circular array antenna. It is pertinent to mention here that the circular array antenna has an important additional advantage of being used on curved surface of a space borne vehicle, too. Owing to the simplicity in circular patch microstrip antenna, we have chosen it as a building block for all the array antennas under investigation.

2. Method of analysis

The configurations of three types of array antennas are shown in Figure 1.

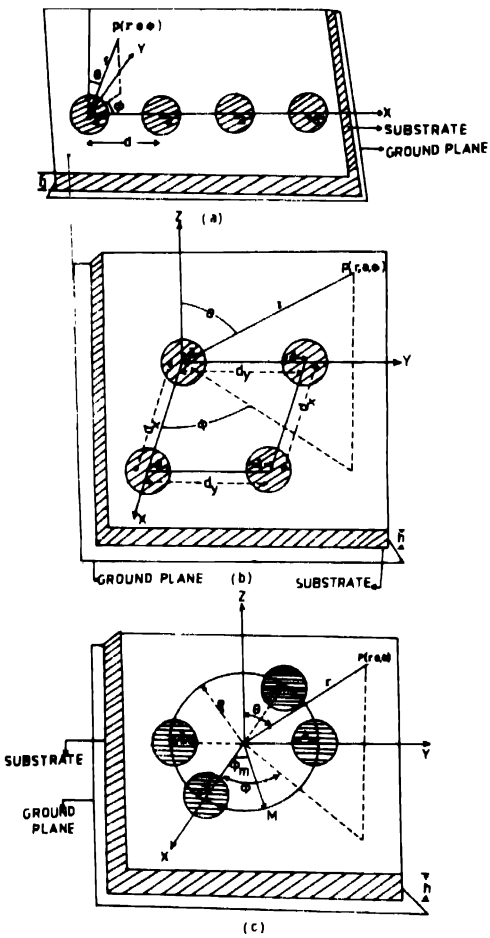

Figure 1. Configuration and coordinate system of four element circular patch microstrip phased array antenna, (a) Linear array, (b) Planar array, (c) Circular array

All the arrays consist of four identical circular patches of radius a on a dielectric substrate of thickness h and substrate permittivity $\varepsilon_r = 3.55$ which is backed by a ground plane. In case of linear array [Figure 1(a)], the elements are uniformly separated by a distance d. However, in Figure 1(b), the elements are arranged in plane and hence element separations are d_x and d_y corresponding to x-axis and y-axis respectively. In circular array *i.e.* in Figure 1(c), the elements are positioned in x-y plane along a circular ring of radius ρ. The elements are taken for point M which moves such that it occupies uniform angular distance ($\phi_m = \pi/2$) between all the four elements from x-axis. Each patch can be excited by a microstrip transmission line connected to the edge or by a coaxial line from the back at the plane $\phi = 0$. Among the various modes that may be excited in such resonators, TM_{nm} mode with respect to Z-axis is considered. Here, n and m are the mode-numbers associated with preferred directions respectively for different array configurations [9–12].

We have developed expressions for the far zone EM-mode and the P-mode components of radiation fields for linear, planar and circular array antennas [9,13,14]. The radiated power into the far-field is determined by integrating the Poynting vector over a large sphere. The expressions for radiation conductance of electromagnetic mode (G_e) and electroacoustic mode (G_p) are obtained as follows :

EM mode :

$$G_e = \frac{2(\beta_e a)^2 I_1}{960\pi} \tag{1}$$

where the integral I_1 is represented by $I_1(L)$, $I_1(P)$ and $I_1(C)$ for the linear, planar and circular array antennas respectively. The expressions for these integrals are given below :

$$I_1(L) = \int_0^{2\pi}\int_0^{\pi} \chi_1^2 \cos^2\{0.5(\beta_e d\cos\theta + \beta_1)\} \times \cos^2(\beta_e d\cos\theta + \beta_1)\sin\theta\, d\theta\, d\phi, \tag{2}$$

$$I_1(P) = \int_0^{2\pi}\int_0^{\pi} \chi_1^2 \cos^2\{0.5(\beta_e d_x \sin\theta\cos\phi + \beta_x) \times \cos^2\{0.5(\beta_e d_y \sin\theta\sin\phi + \beta_y)\sin\theta\, d\theta\, d\phi, \tag{3}$$

$$I_1(C) = \int_0^{2\pi}\int_0^{\pi} \chi_1^2 \left[\sum_{m=1}^{4} \exp j\{\beta_e\rho\sin\theta\cos(\phi - \phi_m) + \beta_1'\}\right]^2 \times \sin\theta\, d\theta\, d\phi, \tag{4}$$

where $\chi_1^2 = \chi_{11}^2 + \chi_{12}^2$, $\chi_{11} = J_n'(\beta_e a\sin\theta)\cos n\phi$,

$$\chi_{12} = \frac{J_n(\beta_e a\sin\theta)}{\beta_e a\sin\theta}\sin n\phi\cos\theta.$$

P Mode :

$$G_p = \frac{30\pi(1-A^2)}{A}\left(\frac{c}{v}\right) I_2. \tag{5}$$

The integral I_2 is taken as $I_2(L)$, $I_2(P)$ and $I_2(C)$ for linear, planar and circular array antennas, difined as

$$I_2(L) = \int_0^{2\pi}\int_0^{\pi} \xi_1^2 \cos^2\left\{0.5\left(\beta_p d \cos\theta + \beta_1\right)\right\} \times \cos^2\left(\beta_p d \cos\theta + \beta_1\right)\sin\theta\, d\theta\, d\phi, \tag{6}$$

$$I_2(P) = \int_0^{2\pi}\int_0^{\pi} \xi_1^2 \cos^2\left\{0.5\left(\beta_p d_x \sin\theta\cos\phi + \beta_x\right)\right\} \times \cos^2\left\{0.5\left(\beta_p d_y \sin\theta \sin\phi + \beta_y\right) \times \sin\theta\, d\theta\, d\phi, \tag{7}$$

$$I_2(C) = \int_0^{2\pi}\int_0^{\pi} \xi_1^2 \left[\sum_{m=1}^{4} \exp j\left\{\beta_p \rho \sin\theta \cos(\phi - \phi_m) + \beta_1'\right\}\right]^2 \times \sin\theta\, d\theta\, d\phi, \tag{8}$$

where $$\xi_1 = \frac{\sin\left(\beta_p h \cos\theta\right)}{\beta_p h \cos\theta} J_n\left(\beta_p a \sin\theta\right) \sin n\phi.$$

Using (1) and (5), the radiation efficiency (η) of the array antenna in plasma medium is defined as

$$\eta(\%) = \frac{G_e}{G_e + G_p} \times 100. \tag{9}$$

The directive gain of a array antenna in a given direction is expressed as

$$D_e = \frac{4\pi M_e}{\int_0^{2\pi}\int_0^{\pi} M_e \sin\theta\, d\theta\, d\phi} = \frac{4\pi M_e}{I_1}. \tag{10}$$

The values of M_e for linear $M_e(L)$, planar $M_e(P)$ and circular $M_e(C)$ array antennas are expressed as

$$M_e(L) = \chi_1^2 \cos^2\left\{0.5\left(\beta_e d \cos\theta + \beta_1\right)\right\} \cos^2\left(\beta_e d \cos\theta + \beta_1\right), \tag{11}$$

$$M_e(P) = \chi_1^2 \cos^2\left\{0.5\left(\beta_e d_x \sin\theta \cos\phi + \beta_x\right)\right\} \cos^2\left\{0.5(\beta_e d_y \sin\theta \sin\phi + \beta_y)\right\}, \tag{12}$$

$$M_e(C) = \chi_1^2 \left[\sum_{m=1}^{4} \exp j \left\{ \beta_e \rho \sin\theta \cos(\phi - \phi_m) + \beta_1' \right\} \right]^2 . \tag{13}$$

Here, notations have usual meaning. The values of D_e are computed for linear, planar and circular array antennas along ($\theta = 3\pi/4$, $\phi = 0$), ($\theta = \pi/2$, $\phi = 0$) and ($\theta = \pi/4$, $\phi = 0$) directions respectively.

3. Results and discussions

We have estimated the three important radiation properties *i.e.* radiation conductance G_e, directive gain D_e and radiation efficiency η for linear [9], planar [13] and circular [14] arrays with varying plasma contents. For a direct and meaningful comparison, we have used the same input values, *viz.* f_r = 10 GHz, ε_r = 3.55, a = 0.47 cm, element separation $d_x = d_y = d = 0.5\ \lambda_0$ and phase excitation $\beta_1 = \beta_x = \beta_y = \beta_1' = \pi/2$ for all the antennas under analysis. A comparison of the results obtained in the present study is given in Tables 1 to 3.

Table 1. Calculated values of radiation conductance, G_e, for microstrip linear, planar and circular arrays with different ratios of plasma-to-source frequency (ω_p/ω_o)

Sl No	ω_p / ω_o	Plasma parameter $A = \left\{1 - \left(\omega_p^2/\omega_o^2\right)\right\}^{1/2}$	G_e (calculated) for 4-element linear array (mho × 10^{-4})	G_e (calculated) for 4-element planar array (mho × 10^{-4})	G_e (calculated) for 4-element circular array (mho × 10^{-4})	G_e (Ref. 6) for 2-element array (mho × 10^{-4})
1	0	1 0	2.0048	2.0164	4.6145	3 1724
2	0 1	0.9949	2.0037	1.9848	4 6635	3.1724
3	0.2	0.9797	2.0996	1 9684	4.9637	3.1724
4.	0.3	0.9539	2.1332	1.8895	5.2813	3.2528
5	0 4	0.9165	2.2095	1 8048	5 4352	3 8038
6	0 5	0.8660	2.2301	1 6441	5 0289	4.0744
7.	0 6	0 8	2.1540	1.4550	3.9340	3.7136
8	0.7	0 7141	1 8549	1.3352	2.5692	3.2626
9.	0 8	0.6	1 2914	0.9521	1 2305	2.8117
10	0.9	0 4358	0.5102	0 5401	0.0685	2.2705
11	0 99	0.1410	0.0079	0.0610	0.0125	—
12	1.0	0	0	0	0	0

It is found (Table 1) that the free-space values of G_e for linear and planar arrays are less than those reported by Saxena *et al* [6] for two element array. However, in case of circular array G_e is greater than that for two element array (Figure 2). In all cases, G_e becomes zero when the value of plasma frequency (ω_p) approaches source frequency (ω_o).

For all other values of ω_p/ω_0, the linear array antenna shows faster drop in G_e as compared to planar and circular array antennas.

Table 2. Calculated values of directive gain, D_e, for microstrip linear, planar and circular arrays with different ratios of plasma-to-source frequency (ω_p/ω_0).

Sl. No	ω_p / ω_o	Plasma parameter $A=\left\{1-\left(\omega_p^2/\omega_o^2\right)\right\}^{1/2}$	D_e (calculated) for 4-element linear array (dB)	D_e (calculated) for 4-element planar array (dB)	D_e (calculated) for 4-element circular array (dB)
1	0	1.0	5 9253	4.8728	3.3709
2.	0.1	0 9949	5.9737	4.9141	3.4062
3	0.2	0.9797	6 0250	4.9117	3 4163
4.	0.3	0 9539	6 1340	5.0264	3 4551
5.	0.4	0 9165	6.2225	5.0658	3 4474
6	0 5	0 8660	6.3993	5.1866	3 4940
7	0 6	0.8	6 4737	5 3533	3.2027
8.	0 7	0 7141	6 5980	5.0163	2 6057
9.	0 8	0 6	6 5974	5.1585	2 2537
10	0.9	0 4358	6.3803	4 8023	1 7558
11	0.99	0.1410	4 9677	3.2916	1 6235
12	1 0	0	0	0	0

Table 3. Calculated values of radiation efficiency, η, for microstrip linear, planar and circular arrays with different ratios of plasma-to-source frequency (ω_p/ω_0).

Sl No.	ω_p / ω_o	Plasma parameter $A=\left\{1-\left(\omega_p^2/\omega_o^2\right)\right\}^{1/2}$	η(%) for 4-element linear array	η(%) for 4-element planar array	η(%) for 4-element circular array	η(%) (Ref. 6) for 2-element array
1	0	1 0	100	100	100	100
2.	0 1	0 9949	72.8973	95.1101	97.7864	89.0909
3	0 2	0.9797	65 1234	90.1336	96 3857	81.8181
4.	0 3	0 9539	58 9867	77.7071	93.4824	72 7272
5	0 4	0 9165	54.0013	66.2352	89.7233	56.3636
6	0.5	0 8660	50.5437	60.9938	84.5743	34.5454
7	0 6	0 8	38.2347	56.2374	78.4326	13.6363
8	0 7	0 7141	27.5348	48.3335	72.5907	4.5454
9.	0.8	0.6	10.6732	40.0039	65.8580	1.8181
10	0.9	0.4358	3.2699	20.1255	57.5041	0
11.	0.99	0.1410	1 2374	2.2354	2.0035	—
12.	1.0	0	0	0	0	—

It is observed from Figure 2 that the value of G_e for linear and circular arrays first increases, then reaches to peak value and then decreases. The peak is observed at $\omega_p/\omega_0 \approx 0.4$ for circular array and $\omega_p/\omega_0 \approx 0.6$ for linear array. A similar peak for G_e was found by Saxena *et al* [6] for two element array at $\omega_p/\omega_0 \approx 0.5$. Thus, above a certain value of ω_p/ω_0 the factor I_1 determining the value of G_e (eq. 1) starts to decrease rapidly.

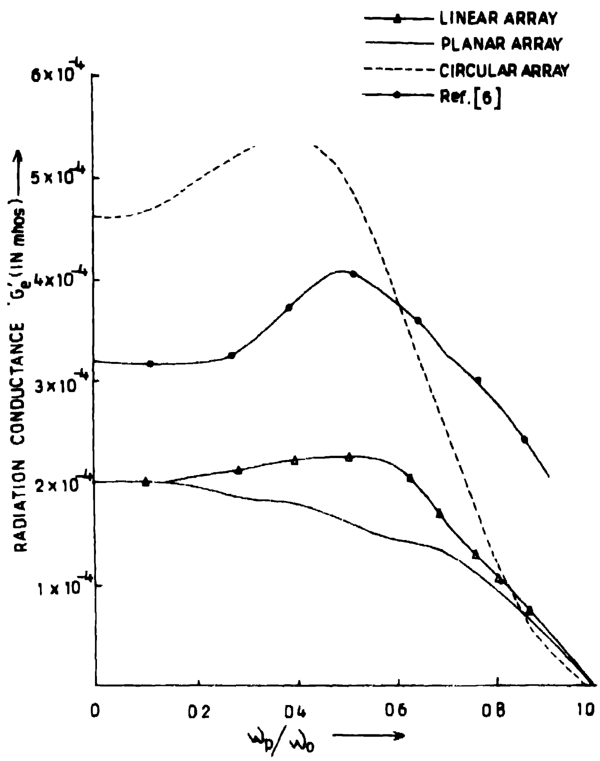

Figure 2. Variation of radiation conductance G_e with plasma-to-source frequency for linear, planar and circular phased array antennas

A comparative study of directive gain D_e for three arrays is made in Table 2. Under the same physical conditions the directive gain of linear array is higher than the corresponding values for planar and circular arrays. The variations of D_e with ω_p/ω_0 are found to differ appreciably from each other for the three arrays. In case of circular array, it is almost uniform up to $\omega_p/\omega_0 = 0.5$ (Figure 3). But for higher values of plasma frequency, it decreases rapidly and ultimately becomes zero for $\omega_p = \omega_0$. For the linear and planar arrays, it is found that D_e first increases and then attains a peak value at a particular value of ω_p/ω_0. Beyond this range, D_e decreases very fast (Figure 3). In case of circular array, the peak is not much pronounced and also the drop of D_e is not as fast as in the other two cases. The peaks found in the plots for D_e seem to be related to the geometrical factors of the arrays. Moreover, it is found that around the peak value of D_e, the rate of increase of M_e is faster than that of I_1 [appearing in eq. (10)] and beyond this value, I_l increases faster than M_e. It is pertinent to mention that the peak in G_e for circular array is more prominent than the peak in D_e for linear array.

Finally, we have compared the radiation efficiency η (%) for the three types of arrays (Table 3). The efficiency decreases in general, as the plasma frequency increases. In

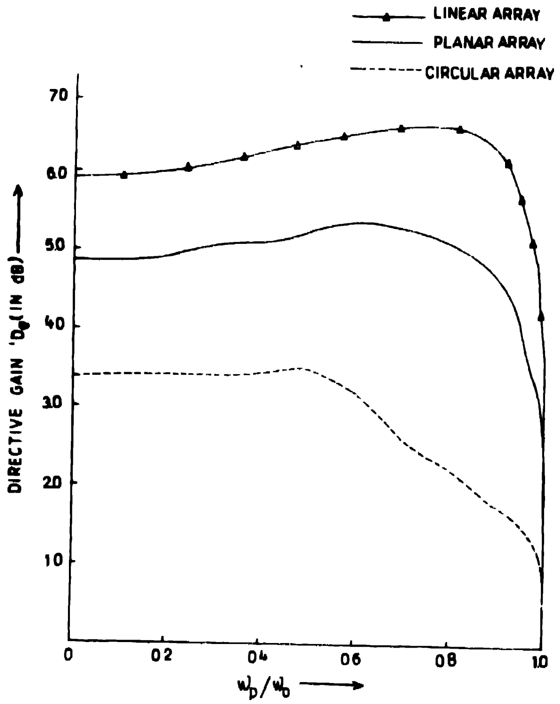

Figure 3. Variation of directive gain D_e with plasma to source frequency for linear, planar and circular phased array antennas.

case of linear and planar arrays, the decreasing trend of radiation efficiency is not uniform, *i.e.* the rate of decrease in different frequency regions is not the same (Figure 4). In case of circular array, the radiation efficiency decreases much slower as compared to the cases of linear and planar arrays. It is remarkable to note from Figure 4 that the radiation efficiency remains 60% or higher for $(\omega_p/\omega_o) \leq 0.8$. This should be considered as a strong point in favour of circular array. Above $\omega_p/\omega_o = 0.8$, the efficiency is found to decrease very fast.

Thus, we have presented a comparative study of important radiation properties of three different arrays. The results obtained in the present study may be useful to find the suitability of the arrays for practical applications corresponding to different plasma frequency regions.

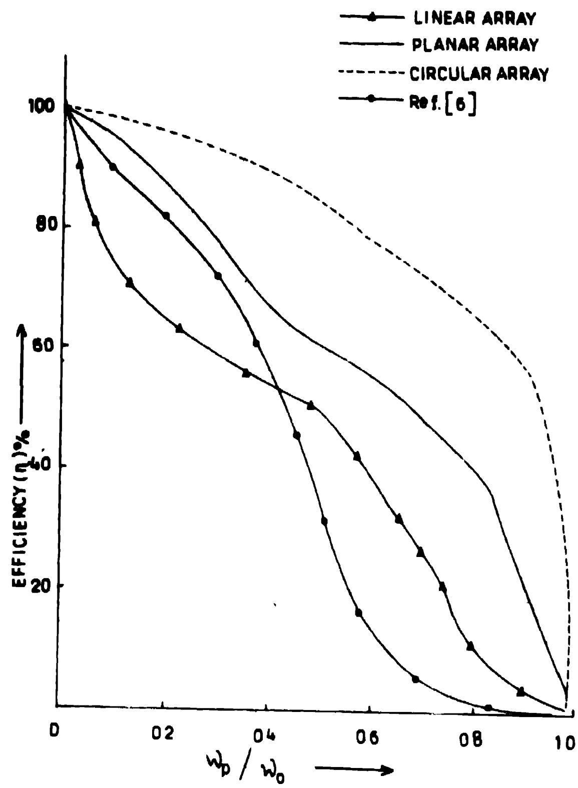

Figure 4. Variation of radiation efficiency η with plasma to source frequency for linear, planar and circular phased array antennas

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